Turbulent Chemical Diffusion in Convectively Bounded Carbon Flames

Daniel Lecoanet, Josiah Schwab, Eliot Quataert, Lars Bildsten, F. X. Timmes,
Keaton J. Burns, Geoffrey M. Vasil, Jeffrey S. Oishi, & Benjamin P. Brown

Abstract

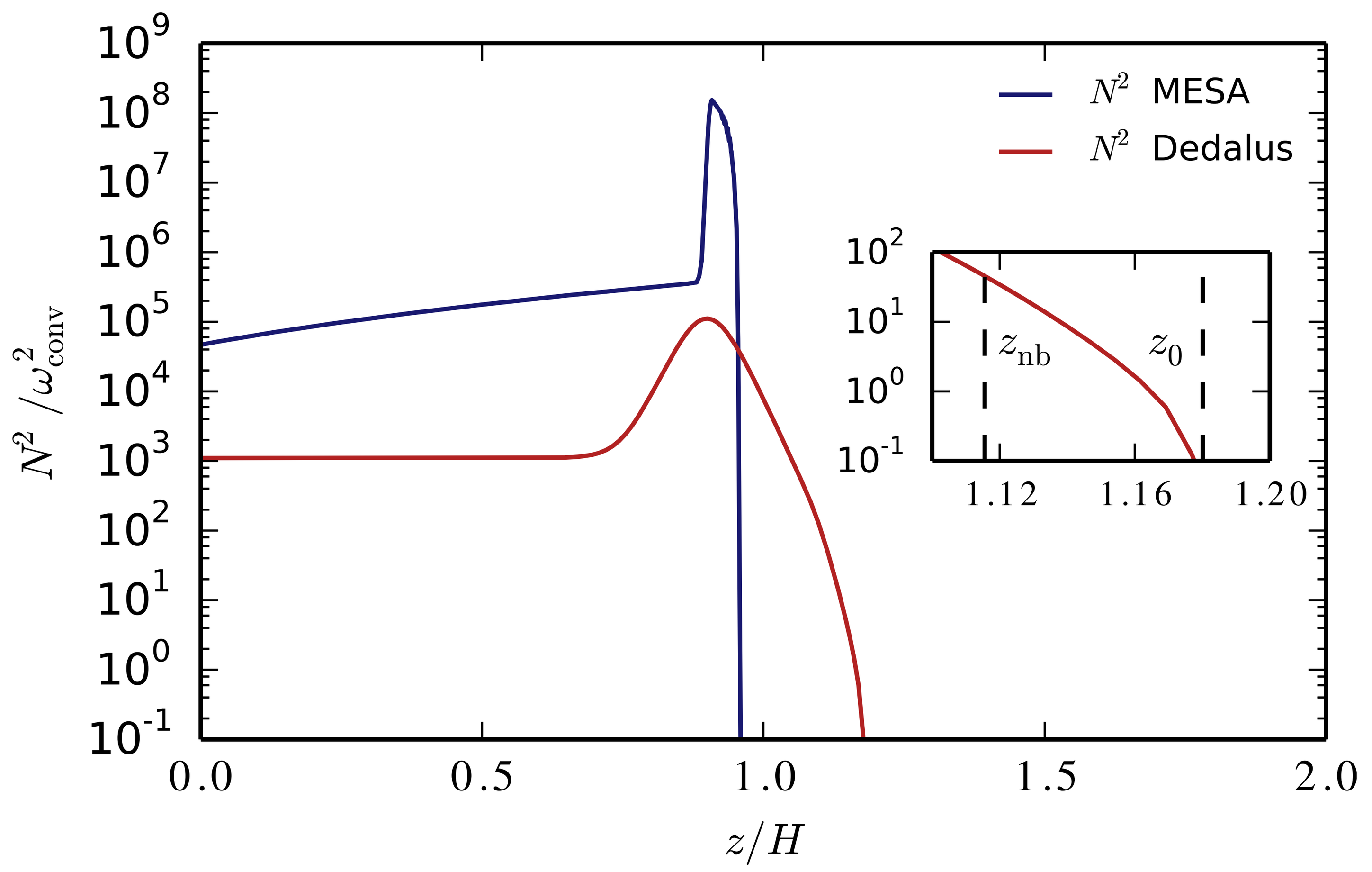
It has been proposed that mixing induced by convective overshoot can disrupt the inward propagation of carbon deflagrations in super-asymptotic giant branch stars. To test this theory, we study an idealized model of convectively bounded carbon flames with 3D hydrodynamic simulations of the Boussinesq equations using the pseudospectral code Dedalus. Because the flame propagation timescale is much longer than the convection timescale, we approximate the flame as fixed in space, and only consider its effects on the buoyancy of the fluid. By evolving a passive scalar field, we derive a *turbulent* chemical diffusivity produced by the convection as a function of height, $D_t(z)$. Convection can stall a flame if the chemical mixing timescale, set by the turbulent chemical diffusivity, D_t, is shorter than the flame propagation timescale, set by the thermal diffusivity, κ, i.e., when $D_t > \kappa$. However, we find $D_t < \kappa$ for most of the flame because convective plumes are not dense enough to penetrate into the flame. Extrapolating to realistic stellar conditions, this implies that convective mixing cannot stall a carbon flame and that "hybrid carbon-oxygen-neon" white dwarfs are not a typical product of stellar evolution.

Equations, Numerics, & Assumptions

We solve the fluid equations in the Boussinesq approximation

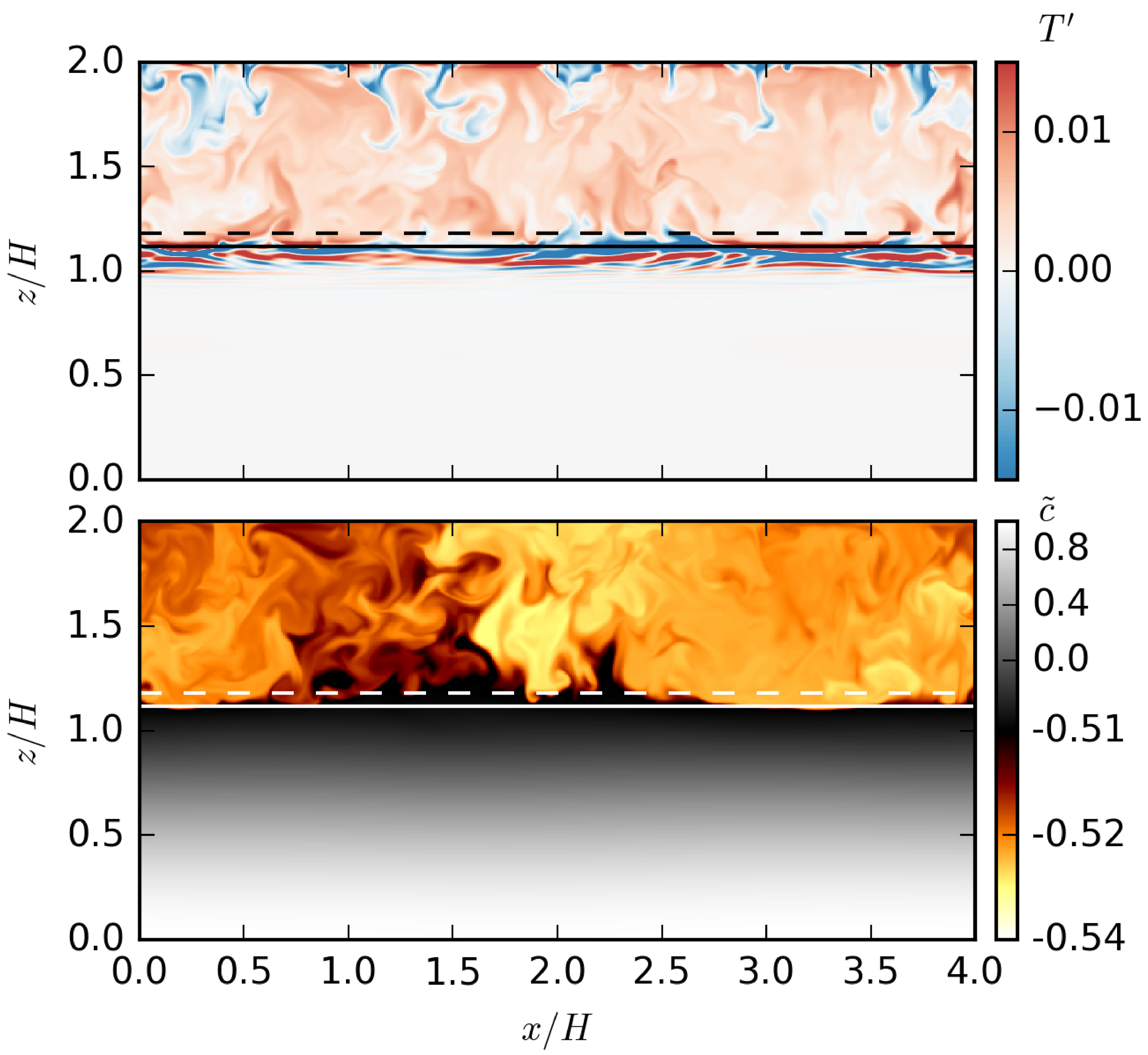
$$\partial_t \boldsymbol{u} + \boldsymbol{\nabla} p - \nu \nabla^2 \boldsymbol{u} - gT\boldsymbol{e}_z = -\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u},$$
$$\partial_t T - \kappa \nabla^2 T = -\boldsymbol{u}\cdot\boldsymbol{\nabla} T + \bar{H},$$
$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0,$$

where $\boldsymbol{u}$ and p are the fluid velocity and pressure, respectively, T is the temperature normalized to a reference value, g is the gravitational acceleration, and $\boldsymbol{e}_z$ is the unit vector in the vertical direction. We neglect the compositional effects on buoyancy (and thus thermohaline mixing), and always use $\nu = \kappa$ for computational convenience. The simulations are initialized with a temperature profile $T_0(z)$ satisfying $N_0^2(z) = g\,\mathrm{d}T_0/\mathrm{d}z$. We include a heating term $\bar{H} = -\kappa\partial_z^2 T_0$ which exactly balances the diffusion of T_0. This maintains the buoyancy profile and convection over the course of our simulations, enforcing the stationary assumption.



The blue line shows the buoyancy frequency squared near a carbon flame from a 9.5 $\mathrm{M}_\odot$ star evolved in MESA. The red line is the buoyancy frequency squared from the Dedalus simulation R8. Due to computational limitations, the buoyancy frequency in the model of the carbon flame is much lower and the transition between the buoyancy peak and the convective region is much more gradual in Dedalus than in the MESA model. These differences both act to enhance the convective mixing via overshoot in Dedalus. The inset shows the neutral buoyancy height $z_{\rm nb}$ and the bottom of the convection zone z_0 in the Dedalus simulation. In the MESA model, this region is not resolved, with a width $z_0 - z_{\rm nb} < 3\times10^{-3}H$.

Sample Simulation Snapshot



Two dimensional vertical slices of the temperature perturbation field (top) and the normalized passive scalar field (bottom) in simulation R9. The color scale for $\tilde{c}$ consists of two linear maps, stitched together at $\tilde{c} \approx -0.5$ to show the small variations within the convection zone. The dashed line shows the bottom of the convection zone, z_0, and the solid line shows $z_{\rm nb}$ the neutral buoyancy height. The perturbations below $z_{\rm nb}$ are waves and yield negligible mixing.

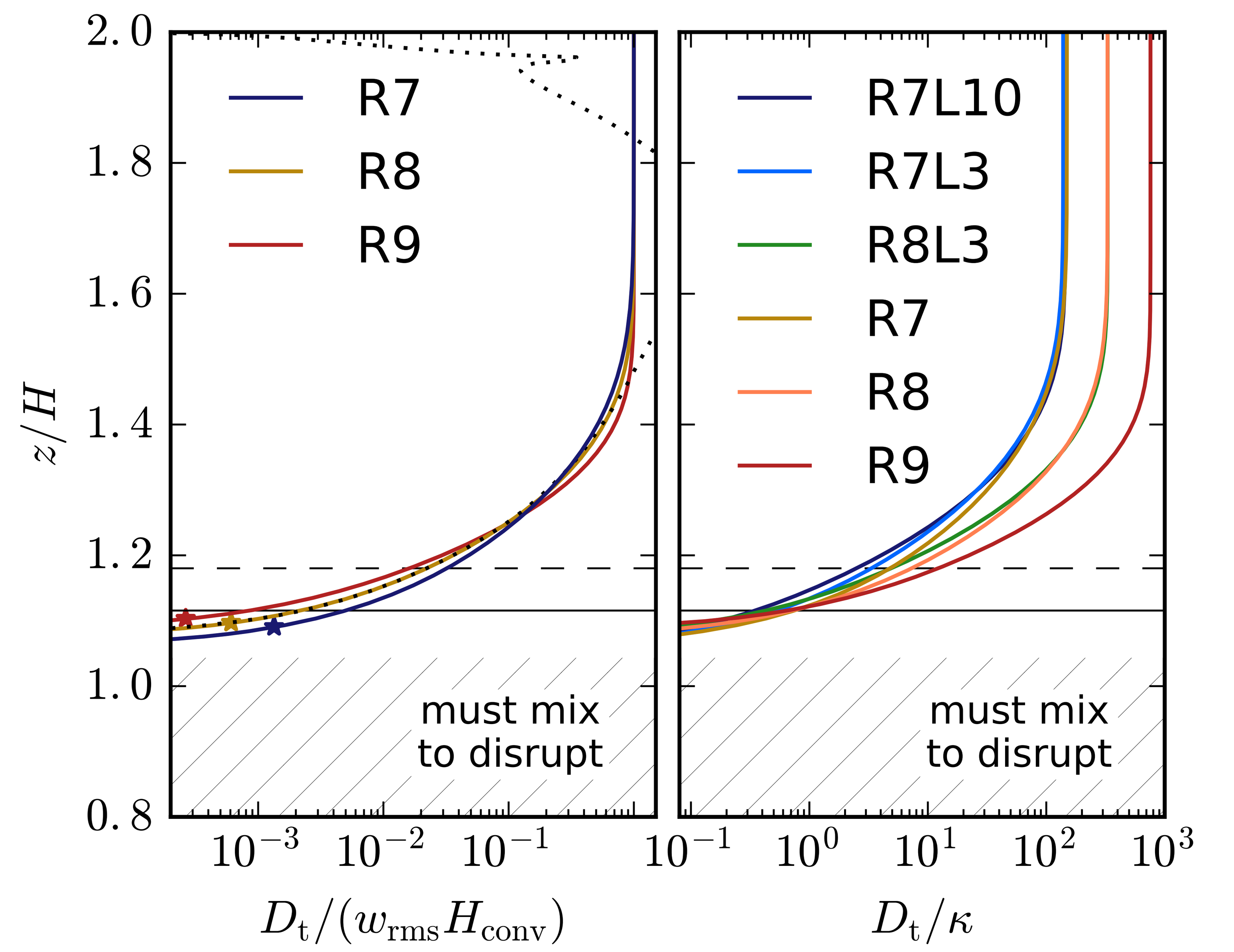
List of Simulations

Name	Ra	Le	Resolution	Re	$D_t=\kappa$	$D_t=0.2\kappa$	$D_t=0$	$L_{\rm ov}$
R7	10^7	1	256^3	150	1.123	1.091	1.066	0.111
R8	10^8	1	256^3	329	1.122	1.097	1.080	0.101
R9	10^9	1	512^3	751	1.122	1.103	1.091	0.090
R7L3	10^7	$10^{1/2}$	256^3	150	1.133	1.094	1.061	0.116
R8L3	10^8	$10^{1/2}$	256^{3a}	329	1.133	1.104	1.083	0.098
R7L10	10^7	10	256^{3a}	150	1.145	1.102	1.063	0.114

[a] The passive scalar field is evolved at 512^3.

The Rayleigh (Ra) and Lewis (Le) number characterize the diffusion in the simulations. The resolution is the number of Fourier or Chebyshev modes used in each direction. The Reynolds number describes the degree of turbulence in the simulation. The three columns after the Reynolds number are the heights at which $D_t = \alpha\kappa$, where $\alpha = 1$, 0.2, or 0. For comparison, in simulation R8, the bottom of the convection zone is $z_0 = 1.180$ and the height of neutral buoyancy is $z_{\rm nb} = 1.116$. The last column is the overshoot length, defined as the distance between the bottom of the convection zone and the location where $D_t = 0$.

Turbulent Mixing in the Simulations



Turbulent diffusivity as a function of height in each of our simulations, both in units of the characteristic convective diffusivity (left panel), and in units of the thermal diffusivity (right panel). We plot a fit to D_t for all simulations, and also plot $|D_t|$ itself in the thin dotted line for simulation R8. The dashed line shows the bottom of the convection zone, z_0, and the solid line shows $z_{\rm nb}$, the neutral buoyancy height. In the left panel, the height at which $D_t = 0.2\kappa$ is marked by an asterisk—mixing can only affect flame propagation above this point. The hatched region shows the region that must be mixed in order to disrupt the flame . Increasing Ra and/or Le causes D_t to approach zero further away from the buoyancy peak.

Conclusions

Our simulations evolve a passive scalar field which heuristically represents the carbon species fraction. Overshooting plumes mix the passive scalar into the convection zone. Our simulations have large diffusivities and are best interpreted as being relevant only in the limit of Rayleigh numbers much larger than are actually computationally achievable.

MESA simulations suggest that a region near the peak of the buoyancy frequency ($N \sim 0.1N_{\rm fl}$) must be mixed with $D_t > 0.2\kappa$ in order to disrupt the flame . None of our simulations show any convective mixing in this region. In all of our simulations, the height at which $D_t = 0.2\kappa$ is well outside the region near the peak of the buoyancy frequency that MESA simulations show must be mixed in order to stall the flame. Moreover, this height shifts closer and closer to the convection zone (away from the flame) as either the Rayleigh number or κ/D (the Lewis number) increase towards more realistic values.

These results strongly suggest that convection provides insufficient mixing to disrupt real carbon flames. The only way out of this conclusion is to posit that for yet higher Ra or Le numbers, the trends we find in mixing with increasingly realistic parameters reverse. Although we cannot rule this out, we regard it as unlikely. Physically, the lack of mixing is due to a simple physical principle: convective plumes must overcome a huge buoyancy barrier to reach the flame. There is no reason to expect them to suddenly be able to do so at even higher Ra or Le. As a result, we conclude that convection provides insufficient mixing to disrupt a carbon flame and that "hybrid C/O/Ne" WDs are unlikely to be a typical product of stellar evolution.